CS598: High-Order Methods for PDEs
Assignment 2 — Due Tuesday, Feb. 9

These exercises are intended to familiarize you with some tools used to derive, analyze, and understand timestepping schemes. Estimated write-up is about two to three pages, including figures.

1. Extend the code `stab_ab3.m` to plot the stability diagram for the explicit timestepping schemes BDF2/EXT2 and BDF3/EXT3. Plot these curves along with the AB3 result.

- At what point does the BDF3/EXT3 curve cross the imaginary axis in the $\lambda\Delta t$-plane? How does this compare with AB3?
- For the 3rd-order scheme, estimate the critical value n_c, the number of gridpoints $n_c = L/\Delta x$ such that the 1D advection problem with 2nd-order centered finite differences will be stable for $\Delta t = .005$, and run your code with n=floor(n_c) and n=ceil(n_c). Give a plot of the solution that demonstates the stable/unstable results.
- For the same test case, demonstrate (e.g., through a plot or table of error vs. Δt) that your code realizes 3rd-order accuracy by perfoming a resolution study. Note that there are two (at least) confounding factors: temporal error during the boot-strapping from timestep 0 and spatial error. Describe how you control these for purposes of your convergence study.

2. Consider the ODE

$$\frac{du}{dt} = f$$

with a given initial condition u^0. For uniform Δt, we can write BDFk/EXTk as

$$\frac{1}{\Delta t}\sum_{j=0}^{k}\beta_j u^{n-j} = \sum_{j=1}^{k}\alpha_j f^{n-j},$$

with the coefficients are given in the table below.

k	β_0	β_1	β_2	β_3	α_1	α_2	α_3
1	1	-1	0	0	1	0	0
2	$\frac{3}{2}$	$-\frac{4}{2}$	$\frac{1}{2}$	0	2	-1	0
3	$\frac{11}{6}$	$-\frac{18}{6}$	$\frac{9}{6}$	$-\frac{2}{6}$	3	-3	1

- Consider a new scheme with an extra β_j and α_j, i.e.,

$$\frac{1}{\Delta t}\sum_{j=0}^{k+1}\beta_j u^{n-j} = \sum_{j=1}^{k+1}\alpha_j f^{n-j},$$

  for $k = 3$. Keep the temporal accuracy at $O(\Delta t^3)$ and use the extra degrees of freedom to find a set (α_j, β_j) that will maximize the stability region on the imaginary axis.
- Test your proposed scheme as above (i.e., w.r.t. stability and accuracy).

(Note — I've never done this myself...it will be a fun exercise for all of us...)