TAM 470 Introduction to Computational Mechanics
Assignment 5

Due: Wednesday, November 16, 2016.

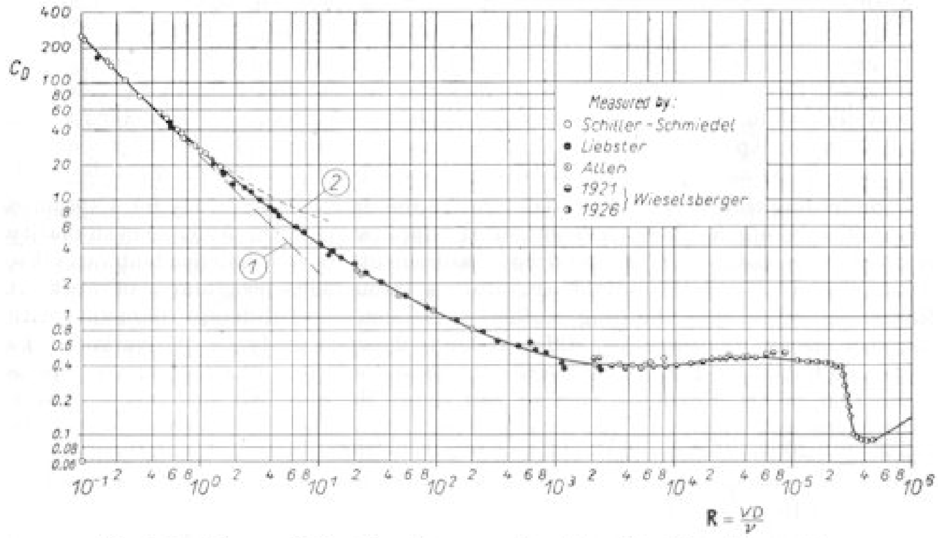

Fig. 1.5. Drag coefficient for spheres as a function of the Reynolds number
Curve (1): Stokes' theory, eqn. (6.10); curve (2): Oseen's theory, eqn. (6.13)

Figure 1: C_D for a sphere versus Reynolds number, Re_D [Schlicting].

This assignment has two short programming problems with some analysis.

For Problem 1, please provide a concise but complete description of the problem and your solution approach, as reporting a carefully done study to someone not familiar with the problem. Include a brief statement of the problem, your numerical approach, your verification procedure (demonstrating to your supervisor that your answer is correct), and your answers. In other words, *this is a writing assignment*, as well as an engineering assignment. You should state where you got your C_D data (but you do not need to resupply the plot). You should clearly state the properties used and the units (MKS, please).

1. Top major league pitchers can throw a 100 MPH fastball. What is the speed (in MPH) of the ball when it crosses home plate? How long (in seconds) does it take to get to home plate?

Procedure: Integrate the 1D equations of motion for a sphere subjected to drag force, F_D:

$$\frac{d}{dt}\begin{pmatrix} x \\ v \end{pmatrix} = \begin{pmatrix} v \\ \frac{1}{m}F_D \end{pmatrix}, \tag{1}$$

with m being the mass of the baseball, x the distance traveled, and v the speed. The drag force is given by

$$F_D = -\frac{A}{2}C_D\rho v^2, \tag{2}$$

with A the cross-sectional area of the baseball ($\pi D^2/4$), ρ the density of air (at 300 K), and C_D

the drag coefficient. The drag coefficient is a function of the Reynolds number,

$$Re = \frac{Dv}{\nu},$$

where D is the diameter of the baseball and ν is the kinematic viscosity of air (at 300K). What is the Reynolds number value for a 100 MPH fastball? (Show this.) C_D can be estimated from the attached figure. What is your value of C_D? (We are not looking for a high-precision estimate of C_D—an estimate from the graph to within 5-10% should suffice.)

All of the necessary quantities (including unit conversions) can be found with a few clicks on google. (Be sure, however, to ask yourself if the dimensions/conversions make sense...) Do all your work in MKS units (for your own sanity), but present the final speed results in MPH.

For 4-credit option. The radar used to measure the pitching speed typically measures it about 10 feet after the ball is released. What is the initial speed (in MPH) of the pitch in this case?

2. Slider Bearing Problem. Use a Galerkin method to solve the 2D incompressible Reynolds lubrication equation

$$-\left(\frac{\partial}{\partial x}H(x,y)\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}H(x,y)\frac{\partial p}{\partial y}\right) = f(x,y) \text{ on } \Omega = [0:L] \times [-\frac{W}{2} : \frac{W}{2}], \tag{3}$$

with

$$f(x,y) := -\frac{1}{2}\rho U \frac{dh}{dx}, \qquad \text{and} \tag{4}$$

$$H(x,y) := \frac{\rho h^3(x)}{12\mu}.$$

We assume that the pressure is ambient ($p = 0$) on the boundary $\partial\Omega$. The relevant parameters in MKS units are:

L	=	.00410 (m)	slider length
W	=	.00089 (m)	slider width
U	=	15 (m/s)	speed of plate
ρ	=	1.225 (kg/m^3)	density of air (@ 300K)
ν	=	15.68e-6 (m^2/s)	kinematic viscosity of air (@ 300K)
μ	=	$\rho\nu$	dynamic viscosity of air
$h(x)$		(m)	height of slider surface as a function of x

A good reference for this problem is:

`http://rotorlab.tamu.edu/me626/Notes_pdf/Notes15%20Gas%20Film%20Lubrication.pdf`

2a. Taper Bearing. Take the slider surface distribution to be the linear taper

$$h(x) \quad := \quad h_0 + \alpha(L - x), \tag{5}$$

with minimum flying height h_0=0.254e-6 m (10 μin $\approx$ 1/4 micron), and pitch angle $\alpha = .001 \times \pi/180$ (in radians).

- Solve for $p(x, y)$ at representative N and plot $p(x, y)$ at N=30.
- Compute the integral of the pressure on the surface to determine the load, F, (force, in Newtons) supported by the bearing for this height distribution.
- Compute the moment of the pressure in the x-direction to determine where the pivot point (i.e., the loading point, pressing down on the bearing) must be if the bearing load is balanced. That is, find the x-component of the center of pressure. Where (what x-value should your pivot be to have a balanced slider?
- The Reynolds equation can easily be solved in 1D if variations with respect to y are neglected. From this exact solution the total load, F, is given in nondimensional form by

$$\frac{h_0^2 F}{6\mu U W L^2} \quad = \quad \frac{1}{(1-r)^2}\left[\ln(r) + 2\frac{1-r}{1+r}\right], \tag{6}$$

  where $r = h_{\max}/h_{\min} = h_{\max}/h_0$. Compare this load to that computed by your code in the case where you have homogeneous Neumann conditions in y (i.e., $\frac{\partial p}{\partial y} = 0$ at $y = \pm\frac{W}{2}$. What is the influence of N (your discretization order) ? (This part is to convince yourself and the reader that your answer is right.)
- As always, defend how you know that your reported answers are correct.

2a. Taper-Flat Bearing. (For 4-credit-hour students.) Take the slider surface distribution to be the taper-flat curve

$$h(x) \quad := \quad h_t(x) \; + \; h_0 \; + \; \alpha(L - x), \tag{7}$$

with minimum flying height h_0=0.254e-6 m (10 μin $\approx$ 1/4 micron), and pitch angle $\alpha = .004 \times \pi/180$ (in radians). Here, $h_t(x)$ is the taper-flat profile that is flat for $x/L \geq 0.1$ and that has a 1 degree taper for $x/L < 0.1$ (like the leading edge of a ski). The profile is provided in `taper_flat_profile.m`.

- Solve for $p(x, y)$ at representative N and plot $p(x, y)$ at N=50.
- Compute the integral of the pressure on the surface to determine the load, F, (force, in Newtons) supported by the bearing for this height distribution.
- Compute the moment of the pressure in the x-direction to determine where the pivot point (i.e., the loading point, pressing down on the bearing) must be if the bearing load is balanced. That is, find the x-component of the center of pressure.
- As always, defend how you know that your reported answers are correct.